Math 2 save the Earth

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1 Introduction

People talk about saving energy and alternative energy sources since decades. Global warming and CO_2 is a relatively new subject for the general public, though experts talked about that one for quite some time as well. Now you know that turning off the light if you do not need it is a good thing, and you might have heard about cars being bad for the environment and planes being even worse, but that little bit of driving to the drugstore every day can not be that bad – or can it? How much CO_2 does that produce, and what does it mean in the general picture?

Fig.1 Traffic in Bangkok [1]

Currently most countries create energy (counting beans: Actually energy can neither be created nor destroyed, only converted from one form into another, e.g. from chemical energy to electric energy) by burning fossil fuels, and/or by using nuclear power plants (which one might think to be CO_2-friendly, yet the production of the nuclear fuel needed for the power plants creates a very large amount of CO_2, and there we don't even start to think about the dangers and the nuclear waste containment). Alternative energy sources like solar energy or wind energy would be a way out. But how much of those would we need to cover our energy consumption? Find out more about how mathematics can help you to help the earth!

2 Curriculum items covered by this unit

2.1 Mathematical curriculum

- Manipulation of terms containing fractions and exponentials
- Percentage
- Interpretation of tables

2.2 Science curriculum

- Electricity
- Energy conversion

3 Tasks and problems

3.1 Energy from the sun

Photovoltaic is used to convert sunlight (mainly the visible part of the spectrum) into electric energy. In Austria, only 0.03% of the energy consumption is covered by photovoltaic. People who are active in environmental protection urge to dramatically increase this kind of energy production, particularly to avoid burning of fossil fuels or nuclear power production. Let us see how much energy can be produced by photovoltaic, and whether it is possible to eventually cover all energy production with solar panels.

Fig.2 Solar panels [2]

Tasks: A standard solar panel is a rectangle with a size of 160 cm x 90 cm. It has a power output of 200 W.

[1] How much area would be covered with solar panels in Austria to cover the total electric power consumption (by 2008, this was 58,884 GWh) by using solar panels? Estimate that the sun shines for an average of 8 hours per day.

[2] Austria has a total area of 83,872 km^2. What percentage of that area would be covered with solar panels?

[3] About 70% of the Austrian power requirement is covered by hydroelectric power plants. How much area of solar panels would be needed to cover the remaining 30%?

[4] Find the data for other countries and solve tasks [1], [2], [3] for those countries!

Solutions:

[1] One solar panel produces 200 W in one hour. If we estimate 8 hours of sunshine per day, the panel would produce 200 W x 8 h = 1,600 Wh per day. In one year (365 days), it would produce 1,600 x 365 = 584,000 Wh = 0.000584 GWh. To cover the total annual consumption on electric power (58,884 GWh), we would need about $\frac{58{,}884}{0.000584} = 100{,}828{,}767 \approx 100$ million panels. As each panel has an area of 160 cm x 90 cm = 1,6 m x 0,9 m = 1.44 m^2, this would result in an area of 1.44 x 100,828,767 = 145,193,424.48 $m^2 \approx 145$ km^2.

[2] $\frac{145}{83{,}827} = 0.0017 = 0.17\%$

[3] 0.0017 x 0.3 = 0.00051 = 0.051 %. These numbers look promising, however there are several drawbacks: a) The sun does not really shine 8 hours a day, b) most of the country is either covered by housing, farmland, forest, or water, all of which could not easily be covered with solar panels, and c) both the price and the amount of solar panels (100 million panels is really a lot; try to find out the costs for one panel) may be prohibitive. But even if only part of this investment would be made, it would mean a tremendous improvement in environmental impact!

3.2 Wind energy

You probably have seen windmills standing somewhere on the coast or on barren land – and we do not talk about the old, wooden mills that were used to grind corn etc. but about modern, concrete-and-steel constructions. From the distance they don't look like much, but if you stand next to one you can see that they are really big – typically around 80 m! Now the last task showed that covering the Austrian energy consumption with solar panels might not work, but how about using wind power instead?

Fig.3 Windmills at the Danish coast [3]

Tasks: A windmill can produce up to 3600 kW of power (counting beans: the actual power output depends on the wind speed; usually windmills work in a range between 4 m/s and 25 m/s).

[1] Suppose the wind blows day and night within the ideal speed of 13 to 25 m/s, which gives the maximum power output of most windmills. How many windmills would have to be constructed to cover the total electric power consumption of Austria?

[2] Obviously the wind does not blow day and night with the same strength. Try to find out the actual wind speed for a place nearby and repeat the calculation!

3.3 CO_2 – How about you

Climate changes and the role of CO_2 in it are heavily discussed since several years. Many books have been written about it, so we will not go into any details here. But do you know how high your own CO_2 emission is?

Task: The CO_2-emmission of a person consists of several parts: shopping, heating (including hot water), traffic, food, electricity, and infrastructure (counting beans: these are only the main categories, but they include the vast majority of personal CO_2 emissions). Use the following tables to find out your own CO_2 emissions!

[1] Shopping

Basic values and added values of CO_2 emissions for shopping in t/year					
	Basic value	Added values			
	Efficient use	Medium use	High use	Very high use	Exclusive use
(A) Housing	0.6	0.3	0.6	0.8	1.4
(B) Car	0.0	0.3	0.5	1.0	1.3
(C) Furnishing	0.2	0.1	0.3	0.5	0.8
(D) Clothing	0.2	0.1	0.2	0.3	0.5
(E) Leisure	0.3	0.2	0.5	0.9	1.8
(F) Personal	0.2	0.3	0.4	0.6	0.8
Total	1.5				

Table 1 CO_2-emissions: shopping

(A) Basic value for 20 m2 per person; add 0.2 t per 10 m2

Townhouse: +0.6 t

Single family house: +0.8 t

Large house: +1.4 t

House built by ecological principles: half values

(B) Car

Motorbike: 0.1 t

Compact car: 0.2 t

Medium car: 0.5 t

Large car/SUV: 0.8 t

Luxury car: 1.3 t

(C) Furnishing: Basic value for ecological furniture

(D) Clothing: Basic value for few pieces of clothes or clothes from ecologic production

(E) Leisure: Hotels, restaurants, movies, operas, sports clubs, …

(F) Personal: cosmetics, books, CDs, gifts, …

For single households, add 0.2 t, for households with 4 or more persons subtract 0.4 t

[2] Heating

CO_2 emissions in kg/m^2 living area								
	Oil heating				Gas heating			
	> 30 years	10-30 years	< 10 years	Low energy	> 30 years	10-30 years	< 10 years	Low energy
Apartment house	56	42	29	14	40	30	21	11
Townhouse	61	46	33	16	44	33	23	12
Single family house	73	56	39	18	52	38	27	14

Table 2 CO_2-emissions: heating

For electricity, double the value for oil

For coal, 25 % more than oil

For wood, 25 % of value for gas

[3] Traffic

For car: 0.2 t per 1000 km

For public transport (bus, train, tram, metro): 0.05 t per 1000 km

For airplanes: 0.4 t per 1000 km

[4] Food

Basic values and added values of CO_2 emissions for food in t/year				
	Basic value	Added values		
	Efficient use	Medium use	High use	Very high use
Meat products	0.0	0.2	0.4	0.6
Dairy products	0.4	0.0	0.3	0.8
Sweets	0.0	0.0	0.05	0.1
Frozen and ready-made food	0.0	0.1	0.3	1.0

Imported foods	0.0	0.1	0.1	0.3
Conventionally grown foods	0.0	0.1	0.1	0.1
Vegetables, fruit, bread (locally grown/made)	0.1	0.0	0.0	0.0
Soft drinks	0.0	0.0	0.1	0.2
Total	0.5			

Table 3 CO_2-emissions: food

[5] Electricity

Normal value: 0.6 t/kWh. An average household uses 3,500 kWh per year. Try to find out your own electricity consumption.

[6] Infrastructure

Those are emissions that are created by the state, cities, communities etc. for general infrastructure, e.g. hospitals, administration, military. It is estimated to be 1 t/year.

It has been estimated that an emission of at most 3 t per year per inhabitant would not create a major change of climate. How about you?

References

[1] http://flickr.com/photos/14149688@N00/231650442, by Gemma Longman (May 14, 2009)

[2] http://flickr.com/photos/79604620@N00/11350061, by Fernando Tomás (May 14, 2009)

[3] http://commons.wikimedia.org/wiki/File:Windkraftanlagen_Dänemark_gross.jpg by Dirk Goldhahn (May 14, 2009)

Recommendations for further reading

- Lagger, I. *Entwicklung von Anwendungsaufgaben zum Thema Energiesparen für das EU-Projekt Math2Earth*, Universität Wien, 2009