Flying an airplane

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1 Introduction

Airplanes take off and land all the time, and the passengers hardly think about how it all works and what's behind all that. Yet the plane has to be fuelled before even taking off – but how much fuel is needed? And the plane needs a certain length of runway to be able to land – but how much? And if a go-around is necessary, what are the conditions for that? Now before you vow to never get in an airplane again, let us see some of the mathematics that pilots need – and rest assured they know all about it!

Fig.1 United Airlines plane shortly after takeoff

2 Curriculum items covered by this unit

- Manipulation of terms containing fractions and exponentials
- Percentage
- Interpreting tables and charts
- Linear interpolation

3 Tasks and problems

3.1 Fuel calculation

The necessary amount of fuel for a commercial airliner consists of several parts that are calculated before each flight. By changing altitude or airspeed the pilot can save some fuel during the flight. Let us consider an example of a flight from Vienna to Dubai with a Boeing 737-800. The minimum amount of fuel consists of:

a) Fuel for taxiing from the parking position to the runway (200 kg).

b) Fuel for the flight from Vienna to Dubai.

c) 5% of b) as spare to compensate for changing winds, lower altitude and an eventual detour because of lightning etc.

d) Fuel for the flight from the destination airport to the alternative airport (if there is bad weather or a blocked runway), done with a standard speed of 380 kts (knots … nautical miles per hour).

e) Minimum remaining fuel (after the landing there has to be fuel left for 30 minutes of flying).

The distance between Vienna and Dubai is 2,450 NM (nautical miles). The alternative airport Muscat is 160 NM away from Dubai. The plane is flying with an average airspeed of 400 kts. The wind is predicted to have a tailwind component of 34 kts. The aircraft has two engines; each is using 1,200 kg of fuel per hour.

Tasks:

[1] How long will the flight take?

[2] How much fuel has to be filled in the tank? (Hint: Calculate the fuel for the flight, then calculate and add the additional fuel for c), d), and e), then add the taxiing fuel from a) to it).

[3] At large airports fuel is pumped into the planes using underground pipes. The amount of fuel is measured in these facilities is measured by volume (litres), not by mass (kg). How much liters of fuel have to be filled in the tanks, if the density of fuel is 0.79 kg/l?

[4] How long does the fuelling process take if the flow rate is 14 l/sec and there were 3.2 tons of fuel in the tanks?

[5] The pilots think of flying lower than planned, because the tailwind component is stronger by 25 kts if they fly 4000 feet lower. Is this option saving fuel, considering the fact that each 1000 ft lower altitude needs 1% more fuel? How much fuel can be saved, if any? (Hint: Add the tailwind speed to the plane speed, and then repeat the calculations of [1]).

[6] The pilots think of reducing the airspeed to save fuel. By how many percent can they reduce the airspeed if the maximum flying time (to avoid delays) is 5 hours 46 minutes? How much fuel can they save by this, considering that 1% reduction in airspeed means 1% less fuel consumption? (Hint: Calculate the airspeed for the given distance, if the flying time is 5 hours 46 minutes, and then calculate the fuel requirements as in [1]).

3.2 Landing

Fig.2 US Airways plane landing on the runway in Seattle

The landing distance of a commercial airliner is influenced by several parameters. In flight preparation, the predicted landing distance is calculated and updated during the flight. The landing distance depends on

a) Total weight of the airplane: sum of weight of the empty plane, estimated weight of passengers (females are assumed to have 76 kg, males 84 kg), weight of luggage, weight of freight, planned weight of fuel at landing
b) Altitude of the airport
c) Wind
d) Slope of the runway
e) Temperature
f) Landing speed of the plane
g) Reverse thrust
h) Surface of the runway
i) Flaps

These parameters are used to calculate the actually required landing distance. An additional 15% (if the runway is dry) or 65% (if the runway is wet) is added to account for imprecision (i.e. late touch down, higher speed, slippery surface) and to create a safety buffer.

We want to calculate the landing distance for a landing with a Boeing 737-800 in Innsbruck. The landing is planned with flaps 40 in western direction. The weight of the empty plane is 35 t, and there are 95 women and 80 men aboard. The luggage weighs 3,300 kg, and the weight of fuel at landing is 2,600 kg. The pilot decides to land with automatic brakes (AUTOBREAK 3), approach speed 10 kts above minimum speed at flaps 40 (vref 40+10), and without reverse thrust (NO REV). The following data is reported from Innsbruck: Dry runway, altitude 2,000 ft, wind 10 kts from western direction (headwind), no slope of runway, temperature 11°C (no need for temperature corrections). The airplane manufacturer published the following table regarding landing distances:

	REF DIST	WT ADJ	ALT ADJ	WIND ADJ PER 10 KTS		SLOPE ADJ PER 1%		VREF ADJ	REVERSE THRUST ADJ	
BREAKING CONFIGURATION	60000 KG LANDING WEIGHT	PER 5000 KG ABOVE/ BELOW 60000 KG	PER 1000 FT ABOVE SEA LEVEL	HEAD WIND	TAIL WIND	DOWN HILL	UP HILL	PER 10 KTS ABOVE VREF40	ONE REV	NO REV
MAX MANUAL	1375	90/-85	35	-60	220	35	-25	110	70	155
MAX AUTO	1495	95/-85	35	-60	225	35	-25	115	75	165
AUTOBRAKE 3	1745	115/-105	40	-75	265	15	-10	185	10	20
AUTOBRAKE 2	2200	165/-155	65	-100	365	30	-35	200	40	40

Table 1 Normal configuration landing distances for a Boeing 737-800 with good reported brakes

Tasks:

[1] Calculate the landing weight. (Hint: Add the passenger, luggage and fuel weight to the weight of the empty plane)

[2] How long is the required landing distance? (Hint: Use the appropriate entries of the table)

[3] Can a landing in Innsbruck be performed? Try to find out the length of the runway in Innsbruck!

[4] How long is the landing distance, if the pilot chooses MAX AUTO breaks? Can a landing in Innsbruck be performed with this setting?

3.3 Go-around (missed approach)

In case a pilot has to perform a go-around, the plane has to achieve a certain climb gradient to be able to safely pass over obstacles. For every possible mode of approach and every runway, a landing manoeuvre is developed and a certain decision height is calculated, so as to gain enough altitude after a go-around and start a new approach or fly to an alternative airport.

We plan an instrument landing with a Boeing 737, considering one engine out, in Salzburg airport on runway ILS 16 in south-eastern direction. The temperature is 22°C, the airport altitude is 1,411 ft, the weight of the plane is 60 t, and the landing is performed with a speed of vref 40+5. The air conditioning remains on during the landing (engine bleed for packs on), the de-icing for engines and wings is not required (engine anti-ice off, wing anti-ice off). The approach chart for Salzburg shows that the decision height for a Boeing 737 (a plane of category C) is 2,030 ft if the climb gradient is 3%, and 2,090 ft if the climb gradient is between 2.5% and 3%. If the climb gradient is lower than 2.5% the plane cannot land in Salzburg.

Tasks:

[1] Calculate the climb gradient, using table 2. Do this by first finding the climb gradient in the first part of the table for the given temperature and altitude (use linear interpolation if the altitude is not in the table), then make a weight adjustment and a speed adjustment according to the other parts of the table.

[2] Can the landing be performed in Salzburg? If yes, what is the decision height (use approach map)?

[3] Can a landing be performed if the weight is 65 t?

[4] What if the de-icing has to be used (engine and wing anti-ice on)?

[5] What could be done to make an approach possible in task [3]?

[6] Try to find other airport charts and solve [2]-[4] with those!

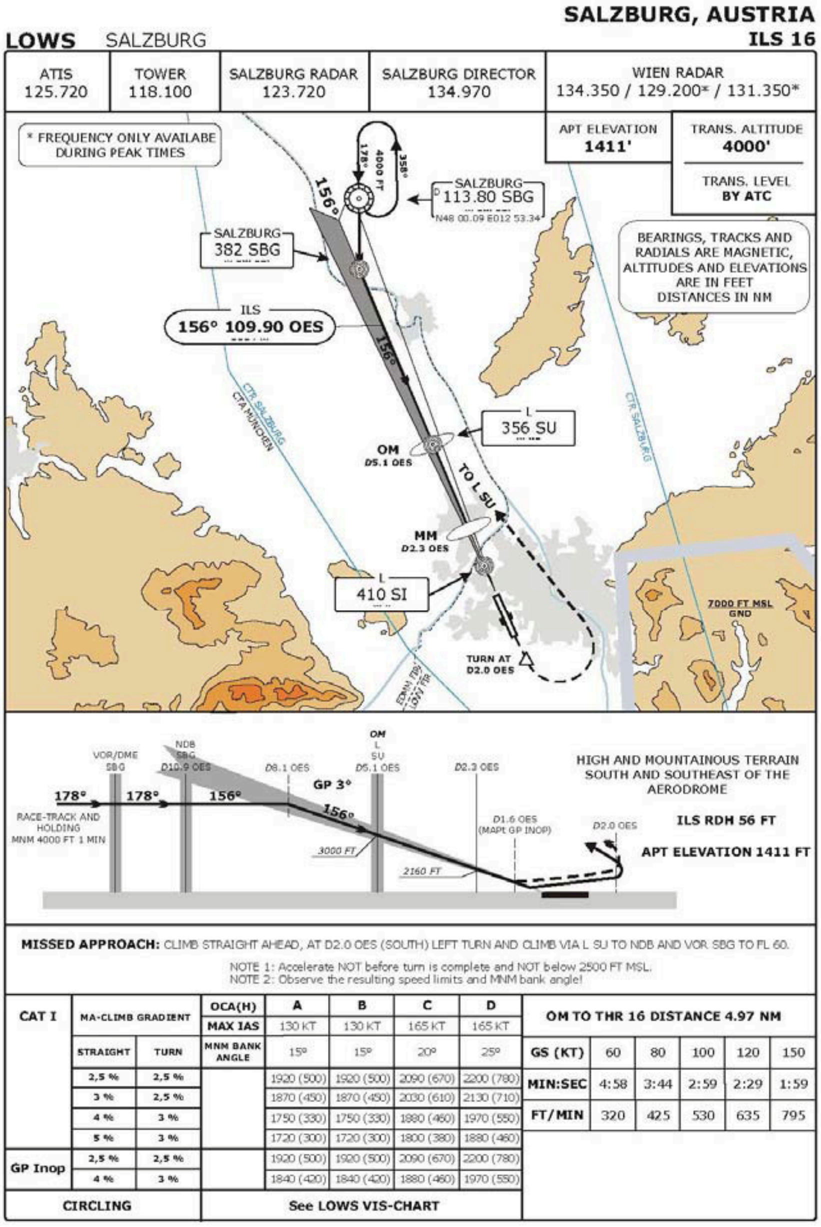

Fig.3 Approach map for Salzburg airport (LOWS)

GO-AROUND CLIMB GRADIENT

Based on engine bleed for packs on and anti-ice off

TEMP °C	REFERENCE GO-AROUND GRADIENT (%)				
	PRESSURE ALTITUDE (FT)				
	0	2000	4000	6000	8000
30	6.27	5.17	4.11	3.00	1.79
26	6.31	5.72	4.63	3.49	2.22
22	6.33	5.74	5.19	4.02	2.69
18	6.36	5.77	5.21	4.51	3.25
14	6.38	5.79	5.23	4.53	3.85

Gradient adjustment for weight (%)

WEIGHT (1000 KG)	REFERENCE GO-AROUND GRADIENT (%)			
	0	2	4	6
65	-2.35	-2.84	-3.36	-3.86
60	-1.72	-2.06	-2.43	-2.79
55	-0.93	-1.13	-1.34	-1.53

Gradient adjustment for speed (%)

SPEED	WEIGHT ADJUSTED GO-AROUND GRADIENT (%)				
	0	2	4	6	8
VREF40	-0.33	-0.35	-0.36	-0.36	-0.36
VREF40+5	0.00	0.00	0.00	0.00	0.00
VREF40+10	0.17	0.18	0.19	0.18	0.18

With engine bleed for packs off, increase gradient by 0.3 %

With engine anti-ice on, decrease gradient by 0.1 %

With engine and wing anti-ice on, decrease gradient by 0.3 %

Table 2 Go-around climb gradient for a Boeing 737-600 with one engine inoperable

Recommendations for further reading

- De Silva, D.A. *Sinnstiftende Aufgaben für den Mathematikunterricht*, Universität Wien, 2008
- Gruber, W. *Unglaublich einfach. Einfach unglaublich*, Ecowin Verlag, Wien, 2006
- Wolke, R. *What Einstein didn't know*, Dell Publishing House, New York, 1999