

Finance matters

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1 Introduction

Sooner or later, everybody has to deal with finances. You pay income tax, but how does this actually work? If your income doubles, does your tax double as well? If you take a loan for your house or your car, you usually pay it back over the years. But sometimes people or companies are not able to pay back in time or at all. How do banks deal with that risk? Be prepared to find out more!

2 Curriculum items covered this unit

- Manipulation of terms containing fractions and exponentials
- Percentage
- Matrices
- Binomial Distribution

3 Tasks and problems

3.1 Loan risk

To estimate the risk of default (i.e. the risk that a customer does not pay back his or her loan), companies are sorted into rating categories according to their creditworthiness. The estimated possible changes in the rating after one year are summarized in a migration matrix.



Fig.1 Standard & Poor's headquarters in New York [1]

When banks give loans to companies or private customers they have to be prepared that some of the customers might not pay the whole sum back, because they are unable to pay or went into bankruptcy. This is called a default. To estimate the risk of default, banks and rating agencies classify their customers into rating categories. Each category corresponds to a certain probability of default. A commonly used classification is the one used by the credit rating agency Standard & Poor's ([1]), using letter codes ranging from AAA (best quality borrowers) to D (has already defaulted and is likely to default again). Simplified versions use letters A to D. Using historic data, rating agencies create migrations matrices describing the probability that a borrower – depending on his current rate – will remain in the same category or change to another category after one year. The element a_{ij} thereby described the probability that a customer who is now in category i will be in category j the following year. A typical migration matrix would look like this:

To From	A	B	C	D
A	89%	8%	2%	1%
B	7%	78%	12%	3%
C	3%	21%	70%	6%
D	0%	0%	0%	0%

Table 1 Migration matrix

Tasks:

- [1] What is the probability for a company in rating category A to be in a lower category next year?
- [2] What is the probability for a company in rating category C to be in the same or a higher category next year?
- [3] Calculate the probabilities for companies in categories A, B, and C to be in category D after two years!
- [4] If the migration matrix for next year is the same as the one for this year, what would the two-year migration matrix be?

3.2 Loans portfolio

A loan portfolio is a set of single – sometimes very different – loans. As we have just seen, these loans have a certain risk of default. If a loan portfolio contains a lot of correlating loans (e.g. many loans from the same branch of industry), there is a high likelihood that many of these loans default at the same time and put the bank at risk. If the diversification is high, the probability for such a concentration of default is much lower. The ideal case is a completely independent loan portfolio.

Task: A loan portfolio consists of 75 single loans with a default probability of 4.5% each. The loans are considered independent.

- [1] What is the probability of 3 loans to default?
- [2] What is the probability of at least 5 loans to default?
- [3] Would this probability be higher or lower if the loans would not be independent?

3.3 Income tax

The amount of the income tax you have to pay depends – of course – on the amount of your income. With most other taxes and fees, doubling the amount to be taxed just doubles the amount of tax. But income tax in most countries is a so-called *progressive tax*, i.e. if you earn double as much money, you usually have to pay *more* than double the amount of income tax. To get the amount of one's income tax, the annual income has to be calculated, i.e. the sum of all annual incomes. From this sum, the income tax is determined (counting beans: Actually there might be parts of your income – e.g. certain bonus payments – with a lower tax rate, and amounts of money that you spent for certain purposes – e.g. gifts to charity – which can be deducted from your annual income, but we will not go into these details, as they vary greatly between states and countries).



Fig.2 “Book of taxes” and headquarters of the Austrian Federal Ministry of Finance [2]

Being an employee in Austria, you normally receive 14 monthly payments a year (for two months, you receive a double payment). For the first 10,000 Euro of your annual income you do not have to pay any tax. For the next 15,000 Euro, you have to pay 36.5%, for the following 35,000 Euro the tax rate is 43.2143%, and for everything above, the tax rate is 50%.

Task: We consider four employees A, B, C, D with a monthly income before tax of 1,000 Euro, 2,000 Euro, 3,000 Euro and 5,000 Euro, respectively.

- [1] Calculate the annual incomes before tax of the employees!
- [2] How much income tax do they have to pay?
- [3] What is the percentage of their annual income that they have to pay for income tax?
- [4] What is their net monthly net income (i.e. the monthly income minus the tax; counting beans: actually in several countries social security contributions are also deducted)?
- [5] Each employee gets a monthly increase of salary (before tax) of 3%. How much does their monthly net income increase (calculate both in Euro and in percent)?
- [6] Each employee gets a monthly increase of salary (before tax) of 200 Euro. How much does their monthly net income increase (calculate both in Euro and in percent)?
- [7] Try to find out income tax rates in your own country and solve tasks [1]-[5]!

Solution:

- [1] A has an annual income of $1,000 \text{ €} \times 14 = 14,000 \text{ €}$, B has an annual income of $2,000 \text{ €} \times 14 = 28,000 \text{ €}$, C has an annual income of $3,000 \text{ €} \times 14 = 42,000 \text{ €}$, D has an annual income of $5,000 \text{ €} \times 14 = 70,000 \text{ €}$.
- [2] A has to pay no income tax for the first 10,000 € and 36.5% for the remaining 4,000 € (now all of the income is covered), meaning A has to pay $4,000 \text{ €} \times 36.5\% = 1,460 \text{ €}$. B has to pay no income tax for the first 10,000 €, 36.5% for the next 15,000 € (now 25,000 € of the income is covered), and 43.2143% for the remaining 3,000 € (now all of the income is covered), meaning B has to pay $15,000 \times 36.5\% + 3,000 \times 43.2143\% = 6,771.43 \text{ €}$. C has to pay no income tax for the first 10,000 €, 36.5% for the next 15,000 € (now 25,000 € of the income is covered), and 43.2143% for the remaining 17,000 € (now all of the income is covered), meaning C has to pay $15,000 \times 36.5\% + 17,000 \times 43.2143\% = 12,821.43 \text{ €}$. D has to pay no income tax for the first 10,000 €, 36.5% for the next 15,000 € (now 25,000 € of the income is covered), 43.2143% for the next 35,000 € (now 60,000 € of the income is covered), and 50% for the remaining 10,000 € (now all of the income is covered), meaning D has to pay $15,000 \times 36.5\% + 35,000 \times 43.2143\% + 10,000 \times 50\% = 25,600 \text{ €}$. We can clearly see that the amount of income tax is not at all proportional to the annual income, but progressive.

- [3] A has to pay 1,460 €, which is $\frac{1,460}{14,000} = 10.4\%$. B has to pay 6,771.43 €, which is $\frac{6,771.43}{28,000} = 24.2\%$. C has to pay 12,821.43 €, which is $\frac{12,821.43}{42,000} = 30.5\%$. D has to pay 25,600 €, which is $\frac{25,600}{70,000} = 36.6\%$.
- [4] This can be done by calculating the monthly tax, i.e. dividing the annual tax by 14 (remember that the employees get their monthly salary 14 times), and subtracting this from the monthly salary. For A, this would be $1,000 - \frac{1,460}{14} = 895.71$ €. Similar calculations can be done with the other employees. Alternatively, you can also subtract the percentage calculated in [3], this would however be not quite as exact due to rounding.
- [5] If A gets an increase of 3%, the monthly income would be 1,030 €, the annual income would be $1,030 \times 14 = 14,420$ €. The income tax would be (see [2] for exact description of the calculation) $4,420 \times 36.5\% = 1,613.30$ €. The monthly net income would then be $1,030 - \frac{1,613.30}{14} = 914.76$ €, and the monthly increase would be $914.76 - 895.71 = 19.05$ €, which would be an increase of $\frac{19.05}{895.71} = 2.1\%$. Similar calculations can be done for B, C, and D.
- [6] Similarly, if A gets a monthly increase of 200 €, the monthly income would now be 1,200 €, the annual income would be 16,800 €. The income tax would be $6,800 \times 36.5\% = 2,482$ €. The monthly net income would then be $1,200 - \frac{2,482}{14} = 1,022.71$ €, and the monthly increase would be $1,022.71 - 895.71 = 127$ €, which would be an increase of $\frac{127}{895.71} = 14.1\%$. Similar calculations can be done for B, C, and D.

References

- [1] <http://www.55water.com>, by Retirement Systems of Alabama (November 13, 2009)
- [2] <http://www.bmf.gv.at>, by Bundesministerium für Finanzen (November 13, 2009)