Space Shuttle Landing

Andreas Ulovec

1 Introduction

A Space Shuttle launch is always an exciting adventure! Countdown, engine ignition, a fiery lift-off, and the Shuttle is away for its next mission to the ISS or some other near earth orbit mission – all covered live by TV stations around the globe.

Fig.1 Space Shuttle *Endeavour* ready to lift off for mission STS-130

However, when the Shuttle (counting peas: actually the airplane-shaped part that lands is called the *Orbiter*, while *Space Shuttle* means the whole compound of the vehicle at take-off, including the two solid-rocket boosters and the orange external fuel tank, all of which is ditched several minutes after take-off.) comes back some days or weeks later, it generates much less interest. And yet it is the landing that is the most challenging part of the flight for the crew – and also the most dangerous. *The Space Shuttle gets very hot while coming back to Earth*, one might read in the internet or the newspapers, but how hot is "very hot"? How long does "coming back to Earth" actually take? Some people have heard about air friction, drag, and insulation tiles, but how does it all come together? Follow us to find out more!

2 Curriculum content covered by this unit

2.1 Mathematics curriculum

- Manipulation of terms containing fractions and exponentials
- Percentage
- Area of plane figures
- Trigonometric functions
- Optional: Derivatives
- Optional: Solving simple differential equations

2.2 Science curriculum

- Air density
- Drag (air friction)
- Energy conversion
- Accelerated movement

3 Tasks and problems

3.1 How hot does a Space Shuttle get?

Before we can answer this question, we first have to see *why* a Space Shuttle (or any other object moving fast through the atmosphere) is getting hot in the first place. The answer is *air friction*. In any kind of friction, kinetic energy is converted into thermal energy. If we think about friction between solids, this is a well known process – rub your hands together, and they get warm, rub faster, they get even warmer. Friction of a solid moving in a gas is a bit different, but the principle of conversion of kinetic energy into thermal energy remains (counting peas: some would say there really is no thermal energy conversion, as increasing temperature just means molecules moving faster, i.e. kinetic energy again). Air molecules (mainly Nitrogen and Oxygen molecules) hit the surface of the solid, and a tiny fraction of the kinetic energy of the solid is converted into thermal energy (or kinetic energy of air molecules). With relatively slow speed this effect is also there, but too small to realize – if you move your hands through the air, you don't feel them (or the air) getting any warmer (counting peas: actually your hands get a bit colder because of vaporization of sweat). At the speed of an airplane, the effect is already measurable. At the speed of a Space Shuttle (about 20 times the speed of a commercial airliner), the effect becomes a major engineering problem – but we will come back to that later.

Task: The most dangerous part of the landing is after the de-orbit burn of a Space Shuttle (i.e. the time when the landing process begins) at an altitude of 122 km and at a speed of $v_1 = 25{,}900$ km/h and the time the Shuttle exits from Radio Blackout (which marks the end of the hottest phase of the flight) at an altitude of 55km and a speed of $v_2 = 13{,}300$ km/h. How much does the temperature of the Shuttle surface (the Shuttles heat capacity is $c \approx 500 \frac{\text{J}}{\text{kg K}}$) increase in this period? (NB: Through an effect called shock wave, only 4% of the converted energy heats up the shuttle, the rest heats up the air and does not concern us here).

Solution:

$$\text{Kinetic energy } E_{kin} = \frac{m \cdot v^2}{2}, \text{ thermal energy } E_{th} = m \cdot c \cdot T$$

Energy conversion: Difference of kinetic energy = difference of thermal energy

Now we take into consideration that only 4% of the converted energy heat up the shuttle:

Difference of thermal energy = 4% of difference of kinetic energy

$$\Delta E_{th} = 0.04 \cdot \Delta E_{kin}$$

$$\not{m} \cdot c \cdot \Delta T = 0.04 \cdot \left(\frac{\not{m} \cdot v_1^2}{2} - \frac{\not{m} \cdot v_2^2}{2} \right) = 0.04 \cdot \frac{\not{m}}{2} \cdot \left(v_1^2 - v_2^2 \right)$$

$$\Delta T = \frac{0.04 \cdot \frac{v_1^2 - v_2^2}{2}}{c}$$

As we are doing the calculations in the metric system, all units (here: particularly the speed) have to be converted into standard units. For speed, this is m/s. The factor of conversion between km/h and m/s is 3.6, i.e. 1 m/s = 3.6 km/h. This leads to $v_1 = 7{,}194$ m/s and $v_2 = 3{,}700$ m/s. Now we can calculate the temperature difference:

$$\Delta T = \frac{0.04 \cdot \frac{v_1^2 - v_2^2}{2}}{c} = \frac{0.04 \cdot \frac{7{,}194^2 - 3{,}700^2}{2}}{500} = 1{,}522$$

The temperature increases by 1,522 °C (counting peas: actually the standard unit for temperature would be Kelvin, but as we are talking about a temperature *difference* here, we can use °C, as degrees Celsius and Kelvin have the same relative scale units). This means that using regular airplane materials for the shuttle surface would not be enough to protect it from the re-entry heat (steel melts at 1530 °C, aluminum melts already at 660 °C). That's why the surface of the shuttle (particularly the parts getting hottest, i.e. the nose cap and the wing leading edges, and to a lesser extend the underside of the main body and of the wings) is covered with a Thermal Protection System – reinforced carbon-carbon at critical places, and insulation tiles made of Silica ceramics that has a high melting point and sheds heat very quickly.

Fig.2 Insulation tiles on the lower side of the wings of Shuttle model *Explorer*

Fig.3 Demonstration of heat shedding by insulation tiles at the *Kennedy Space Center*

3.2 How long is the "hot phase"?

We already know that air friction is the main reason why a Space Shuttle gets hot while landing (counting peas: other, but minor, factors are heat from the sun and radiation). It is also the main reason for the Shuttle getting slower (counting more peas: without the atmosphere, the Shuttle would actually get faster because of gravity, but the vertical acceleration effect of gravity is almost completely countered by the centripetal force at this high speed). Air friction depends on several factors: Speed v (the main factor), air density ρ, (effective) surface area of the object A_{eff}, and the geometric form of the object (described by a form factor, or drag coefficient, c_d). The deceleration is given by

$$a = -\frac{1}{2m} \cdot \rho \cdot v^2 \cdot A_{eff} \cdot c_d$$

Mass, effective area, and form factor of the shuttle can be easily determined and remain fairly constant (counting peas: this would only be exact if the Shuttle flies in a straight line; actually the shuttle makes a couple of S-shaped turns, but this would add too much complexity into the calculation), but the air density depends on the altitude, weather etc. The air density in Earths' atmosphere at a certain altitude h (in m) is given by

$$\rho_h = \rho_0 \cdot e^{-h \cdot 0.00011856}$$

ρ_0 being the air density at sea level (the standard value is $\rho_0 = 1.2250 \frac{\text{kg}}{\text{m}^3}$).

Task: As described above, the "hot phase" of a Space Shuttle landing starts at the end of the de-orbit burn at an altitude of 122 km and at a speed of $v_1 = 25{,}900$ km/h and ends at the time the Shuttle exits from Radio Blackout at an altitude of 55km and a speed of $v_2 = 13{,}300$ km/h. How long does the "hot phase" last? The wing area of a space shuttle is 250 m^2, it is coming in with its nose tilted upwards at about 40°, its mass at landing is approximately 100 t, and the drag coefficient is about 0.078. (NB: For reasonably simple calculations, consider the air density being a constant value $\rho_{55\text{km}}$). Hint: Find the function $v(t)$ giving the relation between speed and time.

Solution: (NB: the equation for speed can either be developed by the students, using derivatives and solving a simple differential equation, or provided by the teacher).

To find relation between speed v and time t, we remind ourselves that deceleration is the change of speed with time, i.e.

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

With the above equation for deceleration by air friction we get

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{2m} \cdot \rho \cdot v^2 \cdot A_{eff} \cdot c_d$$

This is a differential equation of the form

$$\dot{v} + k \cdot v^2 = 0 \text{, with } k = \frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d$$

The solution can easily be found, e.g. by using separation of variables

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -k \cdot v^2 \Rightarrow \frac{1}{v^2} \cdot \mathrm{d}v = -k \cdot \mathrm{d}t \overset{\int}{\Rightarrow} -\frac{1}{v} = -k \cdot t + c$$

$$v(t) = \frac{1}{k \cdot t - c}$$

As we have the border condition $v(0) = v_1$ = 25,900 km = 7,194 m/s we get c = –0.00014, and the equation for speed is

$$v(t) = \frac{1}{\frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d \cdot t - 0.00014}$$

Now we only have to calculate the remaining variables: the air density ρ, and the effective surface area of the object A_{eff} (mass and drag coefficient, as well as the speed at the time the shuttle leaves the "hot phase", are known). As for the air density we made the assumption that this is a constant value $\rho_{55\text{km}}$:

$$\rho_{55\text{km}} = \rho_{55{,}000\text{m}} = \rho_0 \cdot e^{-55{,}000 \cdot 0.00011856} = 0.0018 \frac{\text{kg}}{\text{m}^3}$$

As for the effective area one would think this might be the same as the wing area (250 m^2), but the Shuttle is coming in at an angle of 40° (counting peas: the actual angle varies due to several flight maneuvers performed by the onboard computer, but for most of the re-entry it is indeed 40°), i.e. the effective area must be reduced by the factor sin 40° (if you look at a sheet of paper from a 90° angle you see the full area, if you look at it from another angle the area seems to be smaller):

$$A_{eff} = 250\,\text{m}^2 \cdot \sin 40° = 160\,\text{m}^2$$

Finally we enter all the values into the equation for speed and calculate the time t:

$$t = \frac{1 + v(t) \cdot 0.00014}{v(t) \cdot \frac{1}{2m} \cdot \rho \cdot A_{eff} \cdot c_d} = 3{,}653\,\text{s} \approx 60\,\text{min}$$

The shuttle takes about 60 minutes from the de-orbit burn to the end of radio blackout.

3.3 More about the Space Shuttle

Most missions of the Shuttles are to the International Space Station (ISS), delivering parts, making repairs, and – pretty importantly – shuttle crew members to and from the station. These missions last several days to some weeks. Although the Shuttle is docked to the ISS most of the time, it (and the ISS) is not fixed to a certain point in space, but moves along a given orbit – and pretty fast at that! But how fast? And how many miles does it travel on a mission?

Tasks:

[1] Find out the orbit and speed (relative to earth) of the ISS on the NASA homepage.

[2] Pick any past Shuttle mission to the ISS, and find out how long the mission lasted.

[3] Use this data to estimate how many miles the Shuttle travelled in the mission you picked. Compare this with the data given on Wikipedia.

Fig.4 Space Shuttle model *Explorer*

Recommendations for further reading

- http://www.WhenWillIuseMath.com (December 16, 2009)
- http://www.nasa.gov/audience/forstudents/index.html (December 16, 2009)