

## Maths at the Gym

John Andersen

### 1 Introduction

In mathematics and engineering functions are used to describe motion. This is very important in science and technology. Textbooks describe and calculate motions of imaginary points. The classical Newtonian mechanics is usually introduced as particle mechanics. This seems to be a bit abstract for human beings not used to the ways of thinking in mathematics and physics. “Where am I in the picture of kinematics?” some student may wonder. Understandably enough, since most mathematical descriptions of these phenomena are expressed in a highly impersonal manner. Where is my body when I am doing mathematics and physics?

The aim of this section is to suggest some activities that can personalize aspects of mathematical concepts. Instead of going to the physics laboratory go to the gym or other places where movements of your own body are of central importance. The focus is on describing movements you can immediately relate to, and on the connection between movements and graphs.

Perhaps the right way to start a course in three-dimensional kinematic would be to take a ride on the rollercoaster in the nearest amusement park.

### 2 Jumping boy



**Fig. 1** After a run, a boy jumps athletically over a box

How can you illustrate this event in a way that makes it possible for instance to reconstruct the movement or analyze the motion involved as a whole?

Let your students come up with proposals for this. Probably someone will suggest that you make a movie of the event. Today video recording makes it possible to describe motion in a very direct way.

But when the movie has come to an end, then where is the movement? You see no traces left except what you may have stored in your mind. If you are running through the snow on a winter's day, you leave a trace behind you and a photo can show where you have been – but information of the time is lost.

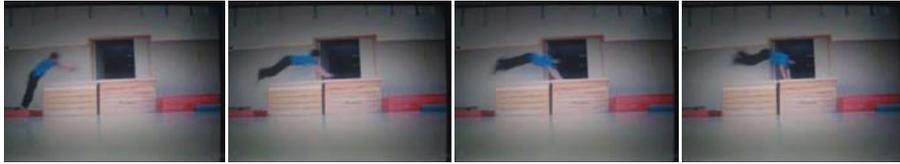


**Fig. 2** My footprints on a snowy day

From this you can see the route, but you cannot see when you actually passed specific points. You need to add information on time. You can stick labels with time on different spots. It is possible.

Watching a film clip live gives the total impression of the run – but what about the details?

You can use still pictures – but if you only concentrate on one picture at a time – for instance when you watch a movie frame by frame – the overall picture is not at hand. But then pick out all frames.



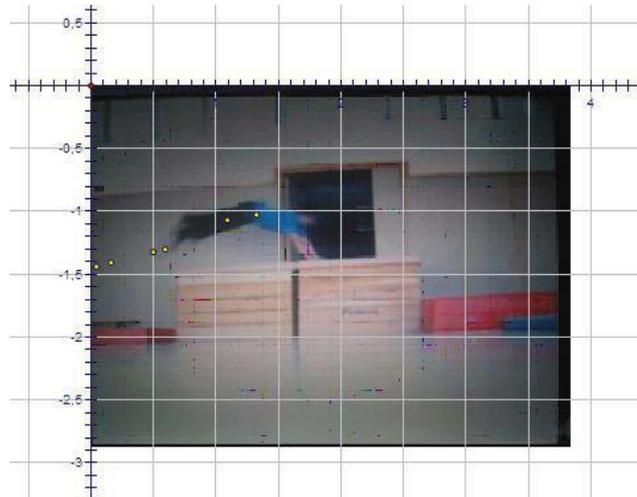
**Fig. 3** Below is shown four of the frames from a video movie of the jumping boy

One can try to stack the photos. It gives a very special impression, and you can really see which points in space are occupied at any time by the jumping boy.



**Fig. 4** The four frames from Fig. 3 put together in one picture.

To simplify we can try to focus on a single point of the boy's body, for instance his waist. Importing the pictures to a geometry programme gives you the possibility to easily measure the coordinates for specific points in each frame.



**Fig. 5** Frame by frame inserted you mark (yellow dots) the position of the boys waist.

Then you can enter the coordinate sets for each selected point into a spreadsheet and draw a graph illustrating the trajectory of the centre of the boy's body.



Fig. 5 Trajectory of the boy's waist now made as an Excel diagramme.

Each point shows the position of the centre of the boy's body (waist) in each frame. The curve is then drawn, but the moments the boy occupies at each point is still not visible. The time points (t) measured in seconds are seen in the table below

t	0	0,08	0,16	0,24	0,32	0,4	0,48	0,56	0,64	0,72	0,8	0,88	0,96	1,04	1,12	1,2	1,28	1,36
---	---	------	------	------	------	-----	------	------	------	------	-----	------	------	------	------	-----	------	------

Table 1 Time (t) in seconds corresponding to the selected datapoints in Fig. 5

You can put each time point as a label at each data point. You will have to do this manually, since the spreadsheet does not offer any procedure for this. Usually another approach is used for this as seen in the graphics below showing time versus measure for each coordinate separately

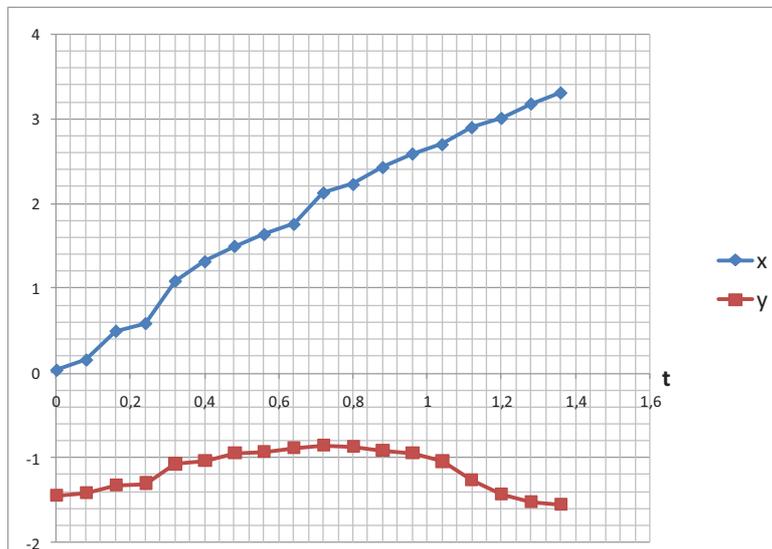


Fig. 6 x- and y-coordinates from Fig. 5 shown as functions of time.

### 3 Transition from real world scenario to traditional graphics

Above is given an example of how you can make the transition from a real world situation to traditional mathematical graphics. You have to realize that a lot is going on in this process. In the real world there is a situation involving movement. As a step towards formalizing this situation you can make a movie of it. The movie introduces a natural way of explicating the timeline. You can import the movie to video editing software (a simple one such as Windows Movie Maker will do in this connection, however, you can buy (or download demo versions of) software such as Dartfish and Data Logger Pro where video editing is integrated with coordinate capture). Now you can have the whole situation dissected into a sequence of moments – each corresponding to a particular time – the frames. Picking a special point from each frame shrinks the whole thing so much that it can be depicted in a single diagram.

What you see is the trajectory of the centre of the moving body. You end up with a picture very far away from the actual scene in the gym. No wonder that a lot of students have difficulties grasping the whole set up if they have no means for making the transition.

#### 3.1 Task

Select some activity in the gym. It can be running, jumping, throwing a ball. Make graphics that describe the motion involved.

### 4 The other way around: Turning graphics/functions into movement

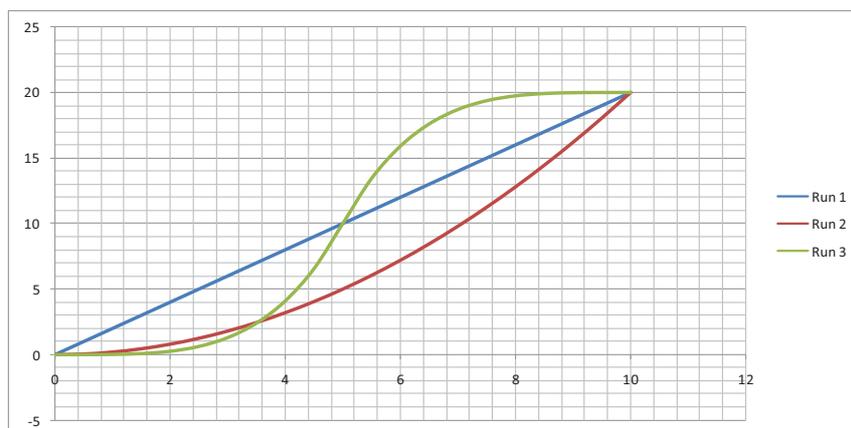
Another way of seeing the problems of connecting function graphs and body movement would be to draw a graph and then move your body in a way that is prescribed by the graph. To move according to a graph you have to mark up on the floor your positions every second (or half second or what may be adequate). Then someone has to direct you, as you move, so that you follow the time flow, for instance by beating a drum.

#### 4.1 Task

##### Experiencing steady movement opposed to accelerated movement

[1] Try to realize the movements depicted in the graphics below. There are three very different runs to be completed. The units are metres and seconds. The third run may be quite demanding in the middle part. If it is too hard, you can modify the graph.

[2] Make other graphs and try them out.



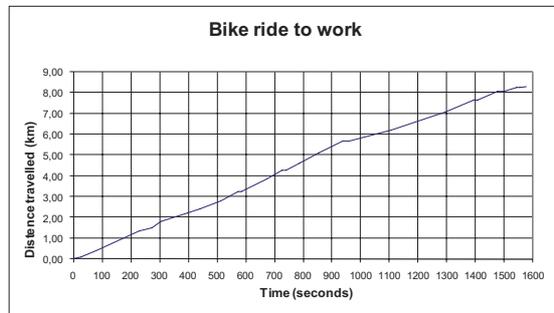
## 5 Riding your bicycle

### 5.1 Task: Describe your bicycle ride from home to school

How would you describe your ride in a way that allows a quick overview of the whole ride?

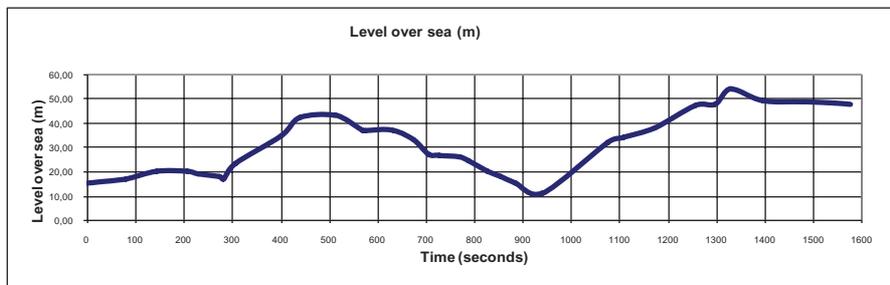
### 5.2 Example

In the actual example I started out with a number of locations along the route, and then I noted down the passing times. If I had had a bicycle computer at the time, I might have noted kilometres and seconds instead. Every description has pros and cons. Listing locations makes it easy to follow my ride on a map. Listing kilometres makes it easy to enter the route into a spreadsheet and print a timetable or a graph. I converted locations into kilometres by using a map source on the internet designed for the planning of jogging routes.



Sometimes one can feel that valuable time is wasted while waiting for traffic signs to change from red to green. Studying a graph like the one above shows for instance that in the overall picture, stops at red do not take up much of the total ride time.

Combining the data for the graph above with a graph showing level over sea gives the next graph.



Working with different functions describing different aspects of the same bike ride can challenge the student to be very careful when interpreting graphs. For instance, do not be misled by the first graph to think that my ride was uphill all the way. It is very different things that are depicted.

If you have access to GPS and data logging equipment, it will be possible to automate the process of graphing actual travelling. The presentation above is to be considered as a first step towards the field of mathematicing motions in the world around you.

### Recommendations for further reading

<http://www.vernier.com/> (January 15, 2010)

<http://www.dartfish.com/> (January 15, 2010)

## Animation – bringing motion into mathematics

John Andersen

### 1 Introduction

If you do traditional textbook mathematics, a great part of your production will be exercises where the outcome takes the form of numbers, drawings or short answers. Evaluation of your production could for instance be comparing results with a list of answers in the exercise book. The products are usually static. Our eyes are designed to detect patterns and movements. Introducing movement to mathematical products may emphasise important aspects of mathematics.

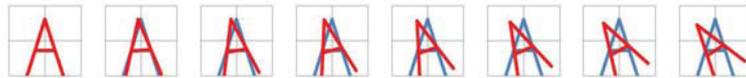
In this section I suggest some activities mainly based on spreadsheet calculations. I have used Excel 2007 to develop the examples. You can do the similar things using dynamic geometry programmes and powerful maths tools such as Maple, MathCAD and Mathematica.

Why use a Spreadsheet such as Excel to do this kind of work? Why not just use any ready-made piece of software? The point is to focus on the connection between numbers, parameters, formulas, functions and graphs without demanding a lot of knowledge about programming and advanced (and expensive) software.

In this short introductory text only a few exemplary cases will be developed. On the project web you will find other and more spectacular examples as well as video shots showing some important techniques.

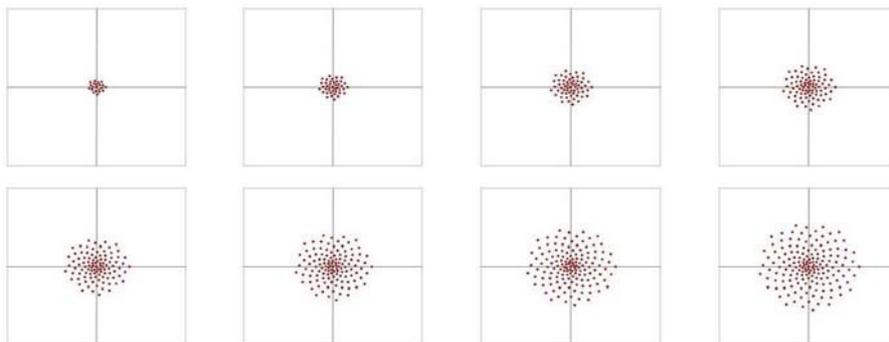
### 2 Animation in this context

When using the term animation we usually refer to a series of pictures (frames) where small, systematic changes in the motive will give the impression of movement when the pictures are shown frame by frame at a sufficiently high speed. Below you see eight frames from an animation of a rotating A.



**Fig. 1** Frames from animating a rotating "A"

And here are frames from an animation of a strongly stylised growth of a sunflower



**Fig. 2** Frames from animating growth of a sunflower

### 3 Prerequisites

Both series above are generated in Excel. The point is that the graphics are generated by tables of coordinate sets. The tables again are calculated by formulas and functions, and into the equations of these formulas are introduced parameters through which you gain control over shape and size. Advancing these parameters step by step enables you to draw the individual frames for the animation.

You should already have acquired elementary spreadsheet skills, for instance by doing economic calculations and drawing graphs.

This text does not contain detailed introduction to the basic principles of using spreadsheets. It will focus on showing some examples and screen captures of formulas and manually advancing calculations one step at a time using the function key <f9>.

Mathematics you have to be familiar with:

- Coordinate geometry (elementary principles for plotting points and graphs from tables).
- Elementary geometry (similarity – scaling figures).
- Vector algebra as far as establishing coordinate formulas for translation and similarity.
- Trigonometric functions and their role in formulas for rotation.

### 4 Examples

A great difficulty in writing about this sort of mathematics in a paper-based text document is that you cannot show how things work live. Therefore this text only provides a few examples as starters. For further examples you will have to visit the project homepage.

#### 4.1: The Letter A - Size regulated by Parameters

Below you see the active cells of the spreadsheet. The size and shape of the letter A are controlled by the parameters in cells B2, B3 and B4. Coordinate tables for the graphics are calculated from the parameters and the formulas in cell area A9:15 are shown in Figure 2.

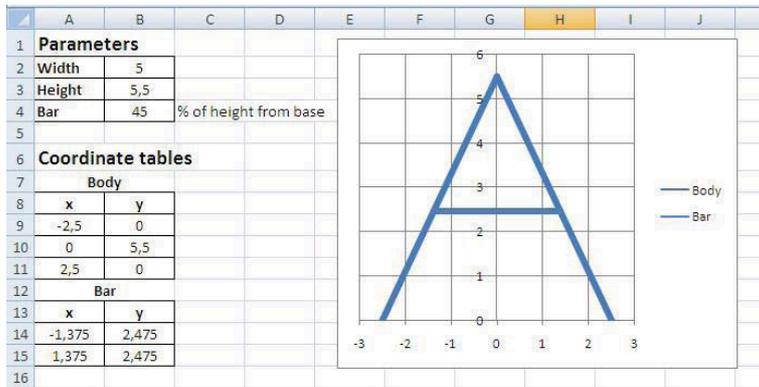


Fig. 3: Active cells in the spreadsheet drawing the letter A

	A	B	C
1	<b>Parameters</b>		
2	Width	5	
3	Height	5,5	
4	Bar	45	% of height from base
5			
6	<b>Coordinate tables</b>		
7	<b>Body</b>		
8	x	y	
9	=-\$B\$2/2	0	
10	0	=\$B\$3	
11	=-A9	=0	
12	<b>Bar</b>		
13	x	y	
14	=A9*(100-\$B\$4)/100	=\$B\$10*\$B\$4/100	
15	=A11*(100-\$B\$4)/100	=B14	

**Fig. 4** Formulas for calculating points for the graphics in Figure 1.

A careful study of the formulas in A8:B15, the parameters in cells B2, B3 and B4 and the resulting picture of the “A” should teach you the idea behind the spreadsheet.

#### 4.1.1 Important technique

##### Handling several series of coordinate tables in the same diagram

If you are used to making spreadsheet graphics by letting the software automatically pick the data areas for graphing, you will have trouble constructing graphics like these because different parts of the figure are based on different coordinate tables. Here is a description of the trick for handling different series of coordinates. On the web page you will find a video showing the steps live.

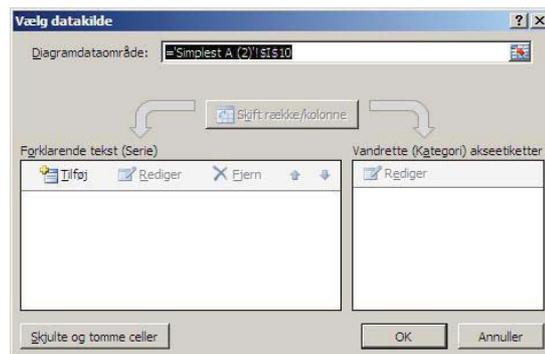
When you have made the part calculating the point coordinates, you are ready for making the graphics.

Choose Scatter (this is the sort of diagrams for making (x,y)-graphs from tables) and click on the icon for piecewise linear.

There appears a diagram area either empty or with some graphics that probably do not meet your actual needs. Right click on the diagram and choose the option Select Data Source



**Fig. 5** Types of graphs



**Fig. 6** Empty Select Data Source Box

Choose Add. You get an input box like the following in Fig. 7

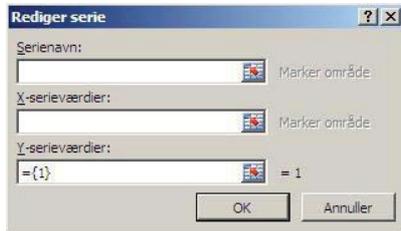


Fig. 7 Empty box for picking data



Fig. 8 Box with data picked for body of "A"

Clicking the red arrows allows you to pick the desired cell areas with the mouse. You can add as many data series as you need. One series for each part of your "A"



Fig. 9 Box with data picked for bar of "A"

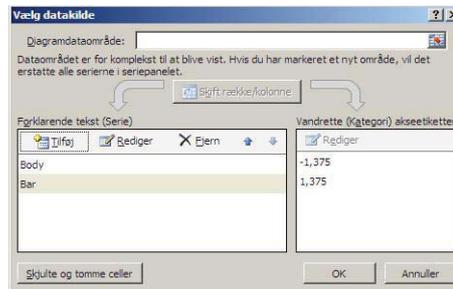


Fig. 10 Box from Fig. 6 with the two series for the "A"

When you have added (or removed) data series, click on OK. Remaining are some editing of the resulting diagram (Fig. 11 below), in order for you to get the picture you want as in Fig. 3 above.

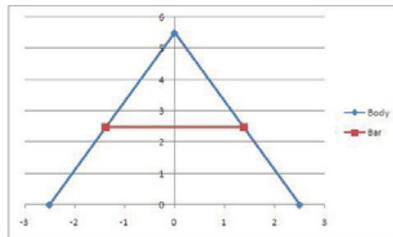


Fig. 11 Preliminary diagram to be edited

### 4.1.2 Tasks

- [1] Carry out the instructions above in an active spreadsheet to make sure that you understand the principles.
- [2] Make other letters and other shapes in the same manner. Experiment with shapes by changing formulas and parameters.

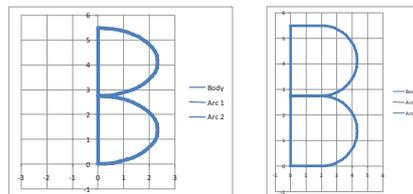


Fig. 12 Two possibilities for shaping a "B"

### 4.2: Zooming the "A" from Example 4.1

The next example shows a spreadsheet designed to zoom the "A". Study the formulas involving the zoom factor. "A" is zoomed down to 75% of full size. Compare with the A in Fig 2.

	A	B	C
1	<b>Parameters</b>		
2	Width	4	
3	Height	5,5	
4	Bar	45	% of height from base
5			
6	Zoom factor (%)		75
7			
8			
9	<b>Coordinate tables</b>		
10	Body		
11	x	y	
12	=-B\$2/2*\$C\$6/100	0	
13	0	=B\$3*\$C\$6/100	
14	=-A12	=0	
15	Bar		
16	x	y	
17	=A12*(100-B\$4)/100	=B\$13*B\$4/100	
18	=A14*(100-B\$4)/100	=B17	

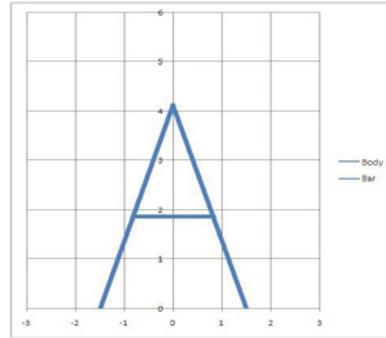


Fig. 14 "A" resized to 75% of the "A" in Fig. 3

Fig. 13 Formulas for zoomin "A"

### 4.3: Dynamically Zooming (Shrinking/Expanding) "A"

Next example makes A shrink gradually from full size to a point and back again. A series of frames shows the dynamics in the spreadsheet diagram.

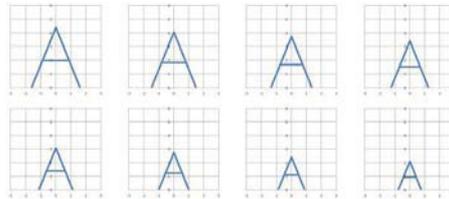


Fig. 15 Frames from animating shrinking "A". See Fig. 16 for formulas used.

	A	B	C	D	E
1	<b>Parameters</b>				
2	Width	4			
3	Height	5,5			
4	Bar	45	% of height from base		
5				Control parameter	Reset parameter
6	Zoom factor (%)		=HVIS(E6=0;100;C6+2*D6)	=HVIS(E6=0;-1;HVIS(C6<=0;1;HVIS(C6>=100;-1;D6)))	1
7					
8					
9	<b>Coordinate tables</b>				
10	Body				
11	x	y			
12	=-B\$2/2*\$C\$6/100	0			
13	0	=B\$3*\$C\$6/100			
14	=-A12	=0			
15	Bar				
16	x	y			
17	=A12*(100-B\$4)/100	=B\$13*B\$4/100			
18	=A14*(100-B\$4)/100	=B17			

Fig. 16 Formulas for shrinking "A"

### 4.3.1 Explanation to cells C6, D6 and E6.

You need to set the calculating mode to manual (see below in section 4.3.2) to make the spreadsheet function as intended. When you press the function key <f9>, parameters and graphics will move. The formula in cell C6 generates the scaling factor. In the actual example it makes the factor vary from 0 to 100 and back again in steps of 2. The formula in cell D6 generates the step direction. When 1 the scaling factor will increase and when -1 it will decrease. The Reset Parameter in cell E6 resets the scaling factor to 100 if E6 contains 0. All other values (I choose 1) make the scaling factor increase with steps  $2 * D6$ .

### 4.3.2 How to dynamically change a parameter step by step

The main point of controlling parameters is described in the following lines: If you type the formula  $=B2 + 1$  into cell B2, you will typical get an error message telling you that a circular reference has occurred. If not the spreadsheet will go on adding 1 to the content of cell B2 ad infinitum if the spreadsheet is in the typical mode of automatically recalculating all formulas whenever something has changed. You can get around this by changing preferences.



Fig. 17 Box for setting preferences for calculations

Setting the Calculating Preferences to Manual and repeated calculations to 1, the spreadsheet will only make a calculation when you press the key <f9>, and the cell B2 now will advance successively through the natural numbers.

Changing the incremental value from 1 to some other number will of course alter this.

## 4.3 Tasks

- [1] Make a shrinking/expanding "B" (or any other letter).
- [2] Make a jumping letter.

## Recommendations for further reading

On the project homepage example spreadsheets and "how to do" videos are available

- <http://www.km.fpv.ukf.sk/math2earth/> (January 2010)



## Mathematical challenges from the town of Galilei

*I observed that even the good artisans fell into the same error as the poets; because they were good workmen they thought that they also knew all sorts of high matters, and this defect in them overshadowed their wisdom.*

**Apology**, by Plato, transl. Benjamin Jowett



 This chapter contains the contributions of the Italian team.

Opening Eyes to Mathematics is one of the main ideas that we try to develop and realise in our project. In this chapter we shall try to give concrete examples how looking around us and trying to model the world around us will give a possibility to prepare and improve math capabilities of pupils, to introduce them to the beauty and excitement of mathematics. Natural inspiration for the Pisa team was the fact that 2009 is the International Year of Astronomy that marks the 400th anniversary of the Galileo Galilei works. The legendary Leaning Tower of Pisa experiment by Galileo shows unlimited possibilities of the human mind to “see” and understand what is around us.

Obviously, there is an essential gap between fundamental discoveries of the Renaissance period and the standard math teaching in High Schools in Italy and Europe now. Somehow our attempt is to show some concrete examples how secondary school teachers can activate pupils creativity and see and recognize around them nice math (and not only math) problems. We are convinced that this can be achieved by means of

- a) application of a group method of teaching and studying: small groups (that we call math labs) of secondary school teachers and pupils work together and possibly use effectively Internet capabilities
- b) work on concrete subject (in our case arguments closely connected with the 400th anniversary of the Galileo Galilei and more precisely with Astronomy and Navigation) and trying to see the world around us and prepare math models and math problems on the subject
- c) possible interaction between these activities making part of the Extra Curricula activities and the standard Curricula ones. We are aware of the difficulties of this interaction, but any small success here can effectively enlarge the group of motivated pupils (and even teachers).

The History keeps for us the legend of the algebraic battles of Niccolò Tartaglia and Antonio Maria Fior, as well as, the competition between Lodovico Ferrari and Tartaglia. The problems, carefully chosen and presented to a Notary, have been printed and then distributed among scientists and researchers. Those works, not only brought immortality to those mathematicians, but also had a great influence on the development of the science.

In attempt to continue this noble tradition, we continue to invite and challenge small groups of secondary school math teachers and motivated students to postulate, solve and develop math arguments closely connected with the 400th anniversary of the Galileo Galilei and more precisely with Astronomy and Navigation. Our scope was not searching for great scientific results or young mathematical stars (we are proud to find any). We just want to involve the students in creating and solving problems, to invite them to share the joy and beauty of math thinking, to feel the excitement of discovering new results starting to look with open eyes around them. We are convinced that the wealth, contained in this part of human knowledge, must belong to all, not only a few chosen.

This part of the book presents the experience of the team of Pisa in organizing small math labs and the results obtained in these labs. We divided this chapter in four sections.

Section 1: Navigation through numbers and fun: Here concrete examples from the Number Theory have been proposed by math labs. The variety of solutions and comparison of the approaches is discussed here.

Section 2: Spherical trigonometry: sum of angles of triangle on a sphere: arguments not very well studied in standard programs and nice problems are introduced. The section is closely connected with the main subject proposed.

Section 3: Astronomy and instruments to draw quadratic curves: concrete math problem and example are combined with the study of the properties and instruments to draw a director circle for a give ellipse. Again this argument is closely connected with the main subject of Astronomy and Navigation and the argument is on the boarder of the standard math programs.

Section 4: From fun problems to deep open problems: here we discuss a problem that seemed to be innocent initially and prepared by the students and the teacher in one of our labs starting from real situation and arriving at deep math problem that have been intensively discussed by the colleagues of the University after the problem appeared and the complete solution is still an open challenging problem.

## Navigation through numbers and fun

Vladimir Georgiev

### 1 Introduction

The main novelty in the approach to adapt the work of our math labs to real life problems and their math contents consists of the problem posing part in the first phase of their activities. More precisely, each team (math Lab) could propose one math problem and the teams were invited to prepare problems coming from the real life. Of special interest were the fields of navigation and astronomy.

One of typical difficulties arises from the fact that there was no requirement on the well-posedness of the problem or on the uniqueness of the solution. The flexibility of the rules in this phase allow to propose very beautiful problems or to treat very interesting situation without knowing a priori that unique solution exists at all.

Each of the teams-labs were formed by groups of 5-7 students and 1 teacher. This specific lab had to invent interesting situation from the real life and to pose a math problem associated with this situation.

Another group of 6-7 math profs from University formed the Commission of the competition and the main task of the Commission in this first phase was to modify the problem if the questions or descriptions were not very clear.

### 2 Curriculum items covered this unit

- Theory of numbers
- Analysis: properties of the sequences
- Combinatorics

### 3 Tasks and problems

We plan to show an example of a problem presented in a very attractive way were one of the math objects used in the problem: sequence of numbers was not defined in a precise way. Formally, the problem is not very well connected with the main argument – Navigation and Astronomy. But this fact shows the difficulty to propose math problems on a give subject. Somehow, the math lab proposing the problem started to prepare the problem as a typical number theory problem and succeeded to have a nice interpretation of the situation as a small fairy tale. We shall present initially the original variant (that needed improvement) and then we shall see why and the improvement was done.

#### 3.1 Problem I: How to find the treasure?

The famous pirate Captain Secante was forced, after terrible tempest, to shore on one unexplored island. While his sailors were trying to repair the ship, he decided to see the island. Just in the centre of the atoll he found a big coffer (lock box), locked by a heavy padlock with unknown combination as well the following brass plate (see fig. 1 below). Below Captain Secante read the following numbers

5	0	123	22
---	---	-----	----

The Captain was more famous (well known) with his pirate raids (attacks) than with his intelligence. So after reading with essential difficulties the text on the brass he decided to try immediately an occasional combination. In the last moment he was stopped by his Boatswain who said: “Wait, Captain! I can help you but only if we divide in equal parts the treasure.” On one of the sides of the coffer the Captain found the text: “This treasure contains golden coins. Their number is 2009!” The Captain without knowing the