Animation – bringing motion into mathematics

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1 Introduction

If you do traditional textbook mathematics, a great part of your production will be exercises where the outcome takes the form of numbers, drawings or short answers. Evaluation of your production could for instance be comparing results with a list of answers in the exercise book. The products are usually static. Our eyes are designed to detect patterns and movements. Introducing movement to mathematical products may emphasise important aspects of mathematics.

In this section I suggest some activities mainly based on spreadsheet calculations. I have used Excel 2007 to develop the examples. You can do the similar things using dynamic geometry programmes and powerful maths tools such as Maple, MathCAD and Mathematica.

Why use a Spreadsheet such as Excel to do this kind of work? Why not just use any ready-made piece of software? The point is to focus on the connection between numbers, parameters, formulas, functions and graphs without demanding a lot of knowledge about programming and advanced (and expensive) software.

In this short introductory text only a few exemplary cases will be developed. On the project web you will find other and more spectacular examples as well as video shots showing some important techniques.

2 Animation in this context

When using the term animation we usually refer to a series of pictures (frames) where small, systematic changes in the motive will give the impression of movement when the pictures are shown frame by frame at a sufficiently high speed. Below you see eight frames from an animation of a rotating A.
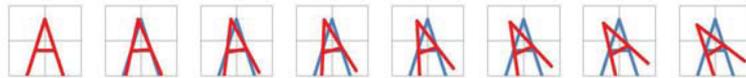

Fig. 1 Frames from animating a rotating "A"

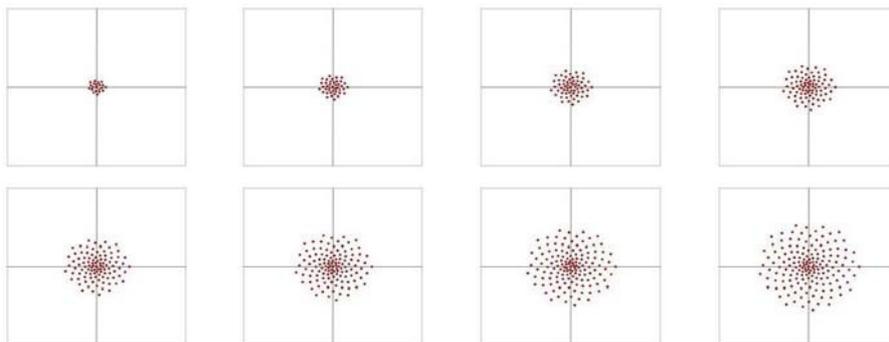And here are frames from an animation of a strongly stylised growth of a sunflower

Fig. 2 Frames from animating growth of a sunflower

3 Prerequisites

Both series above are generated in Excel. The point is that the graphics are generated by tables of coordinate sets. The tables again are calculated by formulas and functions, and into the equations of these formulas are introduced parameters through which you gain control over shape and size. Advancing these parameters step by step enables you to draw the individual frames for the animation.

You should already have acquired elementary spreadsheet skills, for instance by doing economic calculations and drawing graphs.

This text does not contain detailed introduction to the basic principles of using spreadsheets. It will focus on showing some examples and screen captures of formulas and manually advancing calculations one step at a time using the function key <f9>.

Mathematics you have to be familiar with:

- Coordinate geometry (elementary principles for plotting points and graphs from tables).

- Elementary geometry (similarity – scaling figures).

- Vector algebra as far as establishing coordinate formulas for translation and similarity.

- Trigonometric functions and their role in formulas for rotation.

4 Examples

A great difficulty in writing about this sort of mathematics in a paper-based text document is that you cannot show how things work live. Therefore this text only provides a few examples as starters. For further examples you will have to visit the project homepage.

4.1: The Letter A - Size regulated by Parameters

Below you see the active cells of the spreadsheet. The size and shape of the letter A are controlled by the parameters in cells B2, B3 and B4. Coordinate tables for the graphics are calculated from the parameters and the formulas in cell area A9:15 are shown in Figure 2.
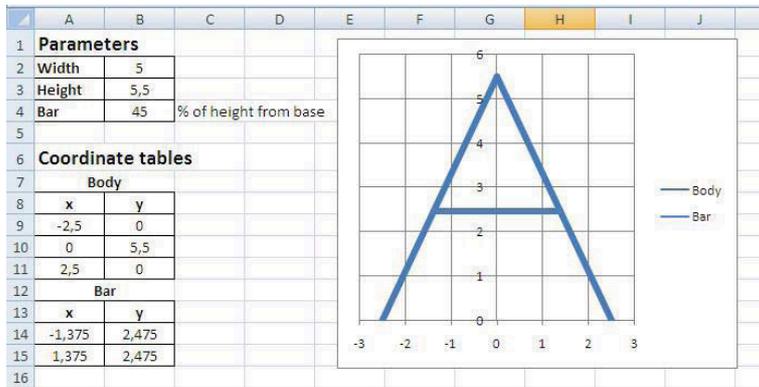

Fig. 3: Active cells in the spreadsheet drawing the letter A

	A	B	C
1	**Parameters**		
2	Width	5	
3	Height	5,5	
4	Bar	45	% of height from base
5			
6	**Coordinate tables**		
7	Body		
8	x	y	
9	=-$B$2/2	0	
10	0	=$B$3	
11	=-A9	=0	
12	Bar		
13	x	y	
14	=A9*(100-$B$4)/100	=$B$10*$B$4/100	
15	=A11*(100-$B$4)/100	=B14	

Fig. 4 Formulas for calculating points for the graphics in Figure 1.

A careful study of the formulas in A8:B15, the parameters in cells B2, B3 and B4 and the resulting picture of the "A" should teach you the idea behind the spreadsheet.

4.1.1 Important technique

Handling several series of coordinate tables in the same diagram

If you are used to making spreadsheet graphics by letting the software automatically pick the data areas for graphing, you will have trouble constructing graphics like these because different parts of the figure are based on different coordinate tables. Here is a description of the trick for handling different series of coordinates. On the web page you will find a video showing the steps live.

When you have made the part calculating the point coordinates, you are ready for making the graphics.

Choose Scatter (this is the sort of diagrams for making (x,y)-graphs from tables) and click on the icon for piecewise linear.

There appears a diagram area either empty or with some graphics that probably do not meet your actual needs. Right click on the diagram and choose the option Select Data Source
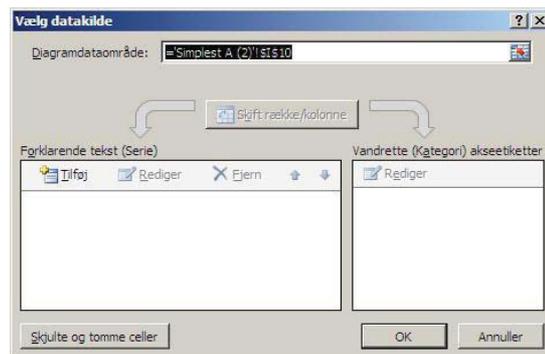
Fig. 5 Types of graphs

Fig. 6 Empty Select Data Source Box

Choose Add. You get an input box like the following in Fig. 7

Fig. 7 Empty box for picking data

Fig. 8 Box with data picked for body of "A"

Clicking the red arrows allows you to pick the desired cell areas with the mouse. You can add as many data series as you need. One series for each part of your "A"

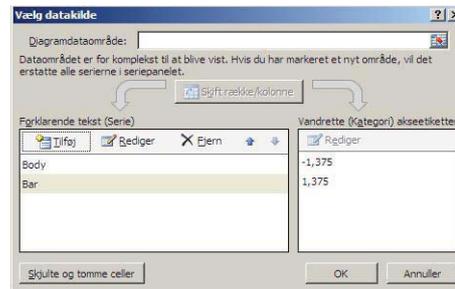**Fig. 9** Box with data picked for bar of "A"

Fig. 10 Box from Fig. 6 with the two series for the "A"

When you have added (or removed) data series, click on OK. Remaining are some editing of the resulting diagram (Fig. 11 below), in order for you to get the picture you want as in Fig. 3 above.
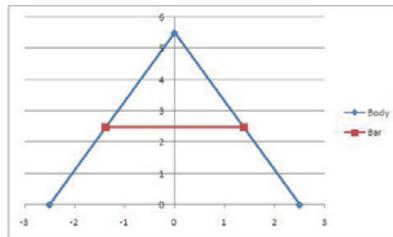

Fig. 11 Preliminary diagram to be edited

4.1.2 Tasks

[1] Carry out the instructions above in an active spreadsheet to make sure that you understand the principles.

[2] Make other letters and other shapes in the same manner. Experiment with shapes by changing formulas and parameters.
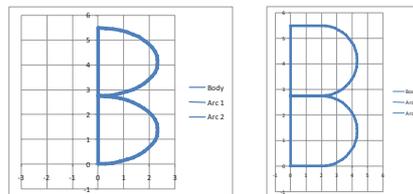

Fig. 12 Two possibilities for shaping a "B"

4.2: Zooming the "A" from Example 4.1

The next example shows a spreadsheet designed to zoom the "A". Study the formulas involving the zoom factor. "A" is zoomed down to 75% of full size. Compare with the A in Fig 2.

	A	B	C
1	Parameters		
2	Width	4	
3	Height	5,5	
4	Bar	45	% of height from base
5			
6	Zoom factor (%)		75
7			
8			
9	Coordinate tables		
10		Body	
11	x		y
12	=-$B$2/2*$C$6/100	0	
13	0	=$B$3*$C$6/100	
14	=-A12	=0	
15		Bar	
16	x		y
17	=A12*(100-$B$4)/100	=$B$13*$B$4/100	
18	=A14*(100-$B$4)/100	=B17	

Fig. 13 Formulas for zoomin "A"

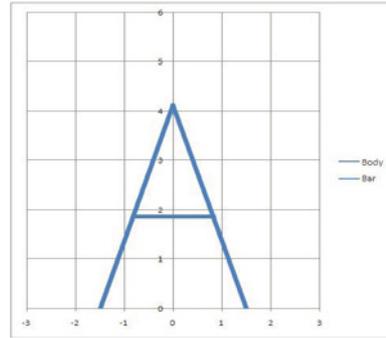**Fig. 14** "A" resized to 75% of the "A" in Fig. 3

4.3: Dynamically Zooming (Shrinking/Expanding) "A"

Next example makes A shrink gradually from full size to a point and back again. A series of frames shows the dynamics in the spreadsheet diagram.
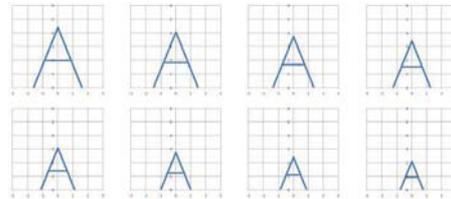
Fig. 15 Frames from animating shrinking "A". See Fig. 16 for formulas used.

	A	B	C	D	E
1	Parameters				
2	Width	4			
3	Height	5,5			
4	Bar	45	% of height from base		
5				Control parameter	Reset parameter
6	Zoom factor (%)		=HVIS(E6=0;100;C6+2*D6)	=HVIS(E6=0;-1;HVIS(C6<=0;1;HVIS(C6>=100;-1;D6)))	1
7					
8					
9	Coordinate tables				
10		Body			
11	x		y		
12	=-$B$2/2*$C$6/100	0			
13	0	=$B$3*$C$6/100			
14	=-A12	=0			
15		Bar			
16	x		y		
17	=A12*(100-$B$4)/100	=$B$13*$B$4/100			
18	=A14*(100-$B$4)/100	=B17			

Fig. 16 Formulas for shrinking "A"

4.3.1 Explanation to cells C6, D6 and E6.

You need to set the calculating mode to manual (see below in section 4.3.2) to make the spreadsheet function as intended. When you press the function key <f9>, parameters and graphics will move. The formula in cell C6 generates the scaling factor. In the actual example it makes the factor vary from 0 to 100 and back again in steps of 2. The formula in cell D6 generates the step direction. When 1 the scaling factor will increase and when -1 it will decrease. The Reset Parameter in cell E6 resets the scaling factor to 100 if E6 contains 0. All other values (I choose 1) make the scaling factor increase with steps 2*D6.

4.3.2 How to dynamically change a parameter step by step

The main point of controlling parameters is described in the following lines: If you type the formula =B2 + 1 into cell B2, you will typical get an error message telling you that a circular reference has occurred. If not the spreadsheet will go on adding 1 to the content of cell B2 ad infinitum if the spreadsheet is in the typical mode of automatically recalculating all formulas whenever something has changed. You can get around this by changing preferences.

Fig. 17 Box for setting preferences for calculations

Setting the Calculating Preferences to Manual and repeated calculations to 1, the spreadsheet will only make a calculation when you press the key <f9>, and the cell B2 now will advance successively through the natural numbers.

Changing the incremental value from 1 to some other number will of course alter this.

4.3 Tasks

[1] Make a shrinking/expanding "B" (or any other letter).

[2] Make a jumping letter.

Recommendations for further reading

On the project homepage example spreadsheets and "how to do" videos are availlable

- http://www.km.fpv.ukf.sk/math2earth/ (January 2010)